SM 3H 2017-18

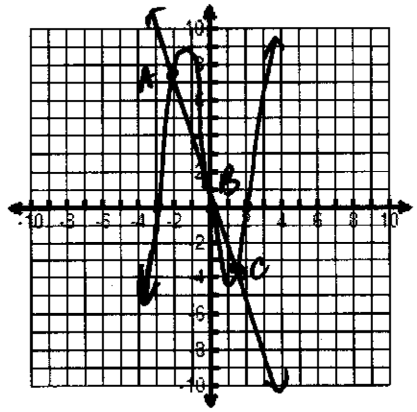
4.1 – 4.4 Review

Name ______________________ **Date** ____________ **Period** ________

Solve the system of equations by graphing. Sketch each function *f(x)* and *g(x)* on one graph and label the intersecting points (solutions).

1. $f(x) = x^3 + x^2 - 6x$
   $g(x) = -3x + 1$

A (-2.17, 7.51)
B (-0.31, 1.93)
C (1.48, -3.44)



Solve each equation by using substitution. Show work!

2. $(x-2)^2 + (x-2) - 6 = 0$

$u = x-2$
$u^2 + u - 6 = 0$
$(u+3)(u-2) = 0$
$u = -3 \quad u = 2$
$-3 = x - 2 \quad 2 = x - 2$
$\boxed{-1 = x} \quad \boxed{4 = x}$

3. $\frac{3}{(x+3)^2} + \frac{5}{(x+3)} = 2$

$u = \frac{1}{(x+3)}$
$3u^2 + 5u - 2 = 0$
$(3u-1)(u+2) = 0$
$u = \frac{1}{3} \quad u = -2$
$\frac{1}{3} = \frac{1}{x+3}$
$\boxed{x = 0}$
$-2 = \frac{1}{x+3}$
$-2x - 6 = 1$
$\boxed{x = -7/2}$

Use sign charts to solve each inequality. Show work!

4. $x^2 + x - 6 \geq 0$

$(x+3)(x-2) \geq 0$ $\boxed{(-\infty, -3] \cup [2, \infty)}$
$x = -3 \quad x = 2$
+ | − | +
-3 2

5. $\frac{x-1}{x^2-36} < 0$

$(x+6)(x-6)$
− | + | − | +
-6 6
$(-\infty, -6) \cup (1, 6)$

Solve each equation for the specified variable. Show all work!

6. $\frac{T^2}{l} = \frac{4\pi^2}{g}$, *solve for g*

$\frac{gT^2}{T^2} = \frac{4\pi^2 l}{T^2}$
$\boxed{g = \frac{4\pi^2 l}{T^2}}$

7. $\left(\sqrt{b^2 - 4ac}\right)^2 = (k)^2$ *solve for c*

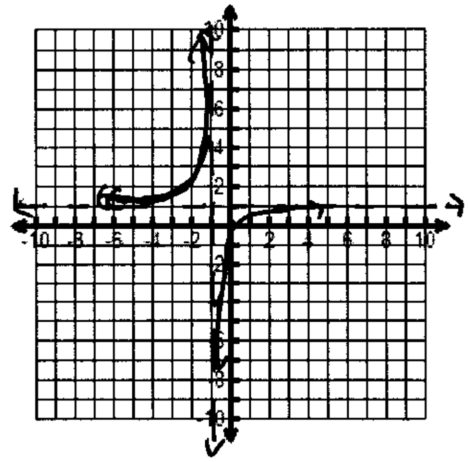
$b^2 - 4ac = k^2$
$\frac{-4ac}{-4a} = \frac{k^2 - b^2}{-4a}$
$c = \boxed{\frac{k^2 - b^2}{-4a}}$

For the sequence write and graph the rational equation that models the relationship between the term and the sequence and its value.

8. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

(n) term	f(n) value
1	$\frac{1}{2}$
2	$\frac{2}{3}$
3	$\frac{3}{4}$
4	$\frac{4}{5}$

$f(n) = \frac{n}{n+1}$



Solve the system of inequalities graphically.

9. $-2x-3y<12$
$3x+5y\leq 15$

$-2x-3y < 12$

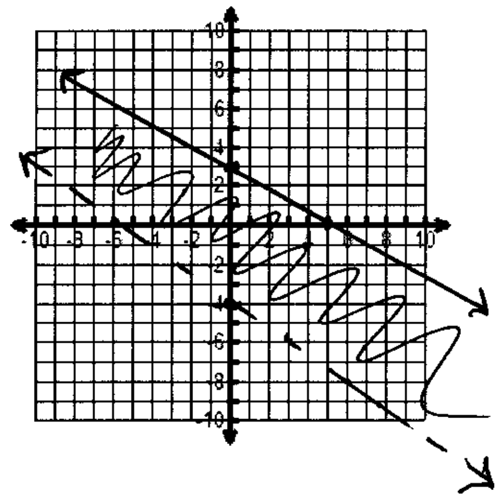
$\frac{-3y}{-3} < \frac{2x}{-3} \frac{+12}{-3}$

$y > -\frac{2}{3}x - 4$

$3x+5y \leq 15$

$\frac{5y}{5} \leq \frac{-3x}{5} \frac{+15}{5}$

$y \leq -\frac{3}{5}x + 3$



Solve the system of inequalities graphically.

10. $x-2y\leq 6$
$2x-4y\geq 0$

$x-2y \leq 6$

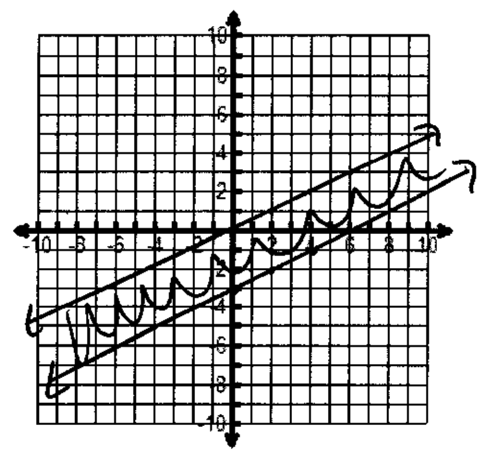
$\frac{-2y}{-2} \leq \frac{-x}{-2} \frac{+6}{-2}$

$y \geq \frac{1}{2}x - 3$

$2x-4y \geq 0$

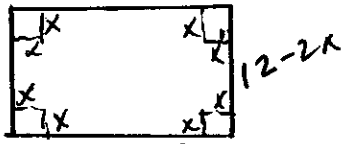
$\frac{-4y}{-4} \geq \frac{-2x}{-4}$

$y \leq \frac{1}{2}x$



11. An open box is made from a rectangular piece of cardboard measuring 20 inches by 12 inches, by cutting identical squares from the corners and turning up the sides. The length of the finished box cannot be less than 12 inches.

a) Draw and label a model of this problem.



b) Write a function for the volume of the box.

$v = lwh$

$v(x) = (20-2x)(12-2x)x$

c) Give the domain of the function in the context of the problem. (use a whole number.)

$12 - 2x > 0$ $\quad x < 6$ $\quad \boxed{(0, 6)}$

$-2x > -12$

d) Give one dimension that the corner squares could have and find the volume for the box.

$x = 1, 2, 3, 4, 5,$ $\quad v(1) = 180\,in^3$ $\quad v(3) = 252\,in^3$ $\quad v(5) = 100\,in^3$

$v(2) = 256\,in^3$ $\quad v(4) = 192\,in^3$

e) Use technology to find the maximum volume the box can have. Give the dimensions (length, width, and height) of the box and the maximum volume, ~~both~~ to the nearest tenth of an inch.

height (x) : 2.4 in $\qquad$ max volume $V(2.4) = 262.7\,in^3$

length $(20-2x)$: 15.2 in

width $(12-2x)$: 7.2 in

12. A banquet hall offers two types of tables for rent: 6-person rectangular tables at a cost of \$28 each and 10-person round tables at a cost of \$52 each. Margret would like to rent the hall for a wedding banquet and needs tables for 250 people. The room can have a maximum of 35 tables and the hall only has 15 rectangular tables available. How many of each type of table should be rented to minimize cost and what is the minimum cost? Show all work!

Number of rectangular tables: 15 $\qquad$ Number of round tables: 16

Minimum cost: \$1252

x - rectangular tables

y - round tables

$C = 28x + 52y$

$x \geq 0,\ y \geq 0$

$x + y \leq 35$

$6x + 10y \geq 250$

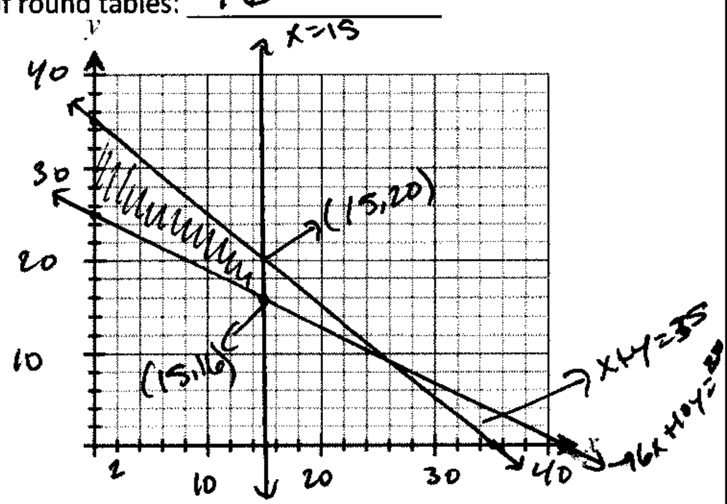
$x \leq 15$

$(0, 25) \rightarrow 1300$

$(0, 35) \rightarrow 1820$

$(15, 20) \rightarrow 1460$

$(15, 16) \rightarrow 1252$



13. Let $P=(x,y)$ be a point on the graph of $y=x^2-9$.

a) Express the distance d from P to the point (0, 1) as a function of x.

$(0,1) \quad (x, x^2-9)$

$$d(x) = \sqrt{(x-0)^2+(x^2-9-1)^2}$$

$$= \sqrt{x^2+(x^2-10)^2} = \sqrt{x^2+x^4-20x^2+100}$$

$$\boxed{d(x) = \sqrt{x^4-19x^2+100}}$$

b) What is d if $x = 0$?

$$d(0) = \sqrt{(0)^4-19(0)^2+100} = \sqrt{100} = \underline{\underline{10}}$$

c) What is d if $x = 2$?

$$d(2) = \sqrt{(2)^4-19(2)^2+100} = \sqrt{16-76+100} = \sqrt{40}$$

$$= \boxed{2\sqrt{10}}$$

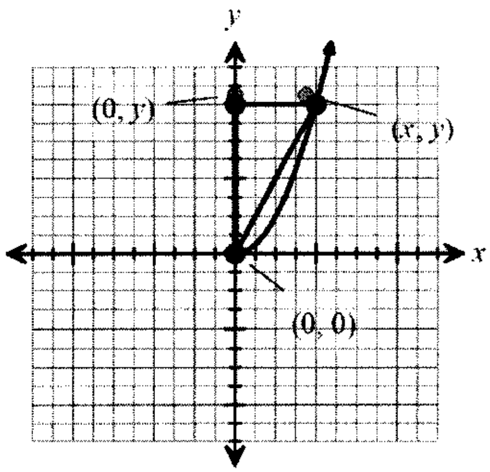
14. A right triangle has one vertex on the graph of $y=x^2$, $x>0$, at (x, y), another at the origin, and the third on the positive y-axis at $(0, y)$, as shown in the figure. Express the area A of the triangle as a function of x.

$A = \frac{1}{2}bh \qquad b = x$

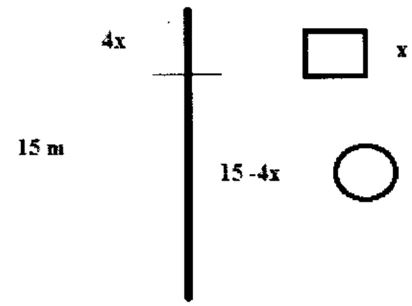
$\qquad\qquad\qquad h = y \to x^2$

$A(x) = \frac{1}{2}x(x^2)$

$$\boxed{A(x) = \frac{1}{2}x^3}$$



15. A wire 15 meters long is to be cut into two pieces. One piece will be shaped as a square, and the other piece will be shaped as a circle. See figure.



a) Express the total area A enclose by the pieces of wire as a function of the length x of a side of the square.

Area of square: x^2

Area of circle : $A=\pi r^2$

$C=2\pi r$

$C=15-4x$ $\qquad \frac{15-4x}{2\pi}=\frac{2\pi r}{2\pi}$

$r=\frac{15-4x}{2\pi}$

Total Area: $x^2+\pi\left(\frac{15-4x}{2\pi}\right)^2$

$A(x)=x^2+\frac{(15-4x)^2}{4\pi}$

b) What is the domain of A?

$15-4x>0$

$-4x>15$

$x<\frac{15}{4}$

$0<x<3.75\,m$

$(0, 3.75)\,m.$